American Lutherie 81 (2005), p. 36-39.

Neck and Bridge Geometry for Domed Top Guitars

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Many luthiers today build "flat top" acoustic guitars with a top that's actually slightly spherically domed – the top is a portion of a sphere rather than being planar. There are a number of reasons for using a domed top - better resistance to humidity-induced cracking, greater stiffness, among others - but the top arching complicates construction in a number of ways. The braces must be arched to the radius of the top, the edge of the sides becomes a wavy curve rather than a planar curve, the kerfing and end and neck blocks need to be angled to fit the arch of the top. In addition, the geometry of the neck and fretboard changes: the top is no longer parallel to the strings as they run over the soundhole to the bridge, but arcs away. This makes the distance from the strings to the top greater at the bridge than at the soundhole, as illustrated in Figure 1. In this side view of an acoustic guitar, the top arching is greatly exaggerated to emphasize the divergence of the top and strings. Note that the fingerboard has been omitted from the diagram, so that the measurement h indicates just the additional bridge height due to the arching of the top.

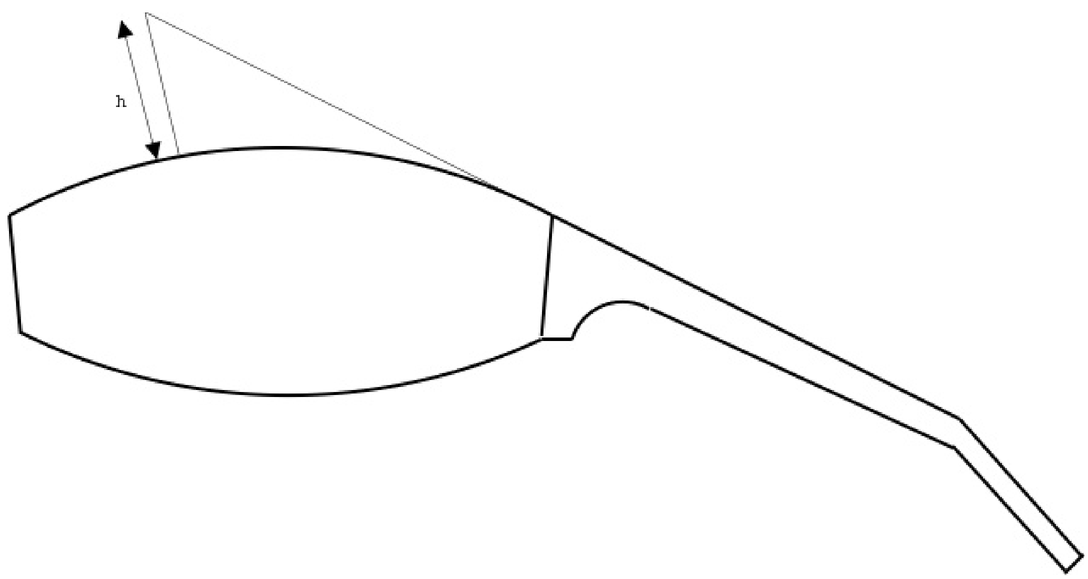

Figure 1

For a true flat-top guitar, the height of the bridge and saddle is easy to calculate: it's just the thickness of the fretboard plus the height of the frets plus the additional height needed to get the desired action. However, because a domed top arcs away from the strings, the bridge height will have an additional component due to the arching of the top. With a little geometry, it's possible to calculate this additional height of the bridge given the radius of the top arch. This can be used either to determine the needed bridge height for a given top arching, or, conversely, to calculate the necessary top arch radius to give a desired bridge height. It is assumed that the top is spherically domed, so that the contour of the top in a side view will be an arc of a circle.

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Neck Surface Tangent to Top Arch

There are two approaches to dealing with the neck and fretboard geometry when using an arched top. The first is a simplified approach that assumes the fretboard and neck surface are tangent to the top arch. This is illustrated in the guitar side view in Figure 2.

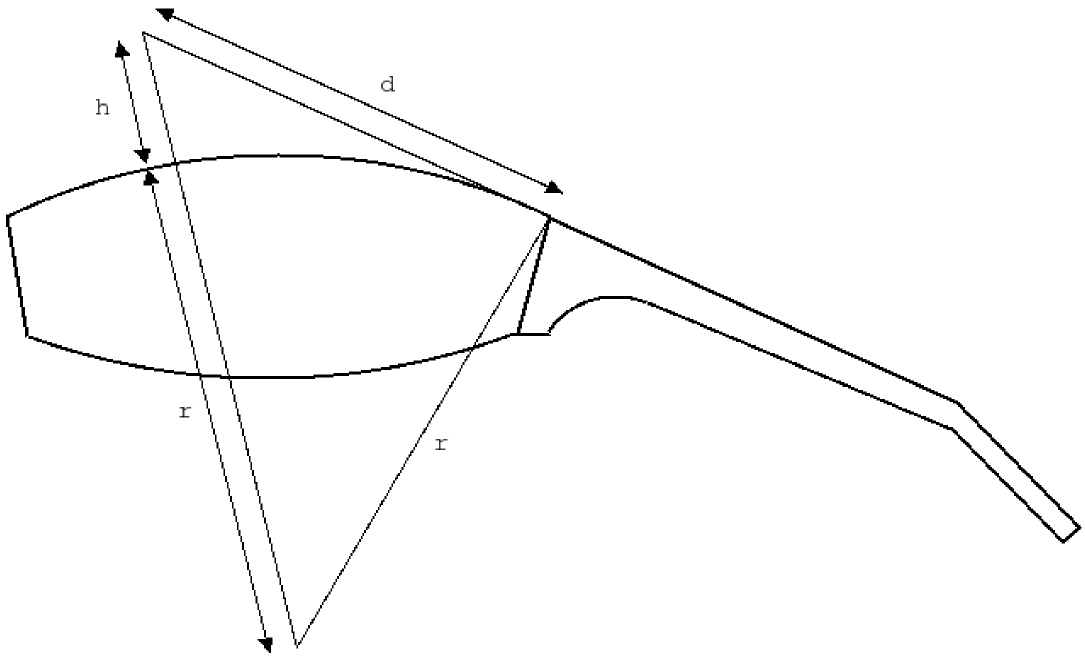

Figure 2

In this figure, the top arching has again been greatly exaggerated to emphasize the geometry. The neck line is extended tangent to the top arch to a point above the bridge. The height of this extended line is labeled as h in the diagram. The radius of the top arch is labeled as r, and the distance from the neck-body junction to the bridge is labeled as d. Since the neck surface is tangent to the top arc, the radius line drawn from the neck-body junction to the center of the top-arch circle is perpendicular to the neck surface. This makes the triangle in the diagram a right triangle, which allows us to use the Pythagorean Theorem:

$$r^2 + d^2 = (r+h)^2$$

or, solving for h, we get

$$h = \sqrt{r^2+d^2} - r = r * \left[\sqrt{1+\frac{d^2}{r^2}} - 1 \right]$$

For necks attached at the 14th fret, the distance d from the neck-body junction to the saddle point is a constant fraction of the guitar scale length:

$$d = .44545 * L$$

For necks attached at the 12th fret, the distance d is just half the scale length:

$$d = .5 * L$$

Thus we can tabulate the height h as a function of the top radius r and the scale length L for both 12-fret and 14-fret necks. Note that these heights don't include the fretboard thickness or fret heights - it's just the additional height at the bridge that's the result of the top curving away from the strings, i.e., it's how much higher the saddle will have to be than it would on a guitar with a truly flat top. The tables below thus tabulate the height of the neckline at the bridge location for various scale lengths and top arch radii; the radii selected are those corresponding to arched workboards available from lutherie supply shops.

Neck joined at 14th fret: height at bridge in inches

	Top arch radius:				
Scale length:	**180" (15')**	**240" (20')**	**300" (25')**	**336" (28')**	**360" (30')**
24.5"	.33	.25	.20	.18	.17
24.75"	.34	.25	.20	.18	.17
25.0"	.34	.26	.21	.18	.17
25.25"	.35	.26	.21	.19	.18
25.5"	.36	.27	.21	.19	.18
25.75"	.37	.27	.22	.20	.18
26.0"	.37	.28	.22	.20	.19

Neck joined at 12th fret: height at bridge in inches

	Top arch radius:				
Scale length:	**180" (15')**	**240" (20')**	**300" (25')**	**336" (28')**	**360" (30')**
24.5"	.42	.31	.25	.22	.21
24.75"	.42	.32	.26	.23	.21
25.0"	.43	.33	.26	.23	.22
25.25"	.44	.33	.27	.24	.22
25.5"	.45	.34	.27	.24	.23
25.75"	.46	.35	.28	.25	.23
26.0"	.47	.35	.28	.25	.23

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Neck Surface as Secant to Top Arch

The above is a simplification because it doesn't consider how the fretboard is attached to the top. As the diagram above shows, the neck line and top arc begin diverging immediately from the point where the neck joins the body. The portion of the fretboard that's glued to the guitar top would then either follow the curve of the top, causing the action over this portion of the fretboard to become large, or would have to be shimmed with a wedged shim to keep the fretboard flat, to compensate for the curved top. In practice an easier approach is just to plane a small flat on the guitar top where the fretboard will be glued. The fretboard can then be glued directly to the top, without requiring a shim, and still be flat from end to end. In this case, the neck line is no longer tangent to the top arc, but instead cuts across it where the fretboard flat has been planed. Figure 3 illustrates the two situations; note that on the left the fretboard is cantilevered out over the curved top surface, while on the right the fretboard lies flush on the flat planed on the top.

Figure 3: Fretboard attachment without and with flat on top

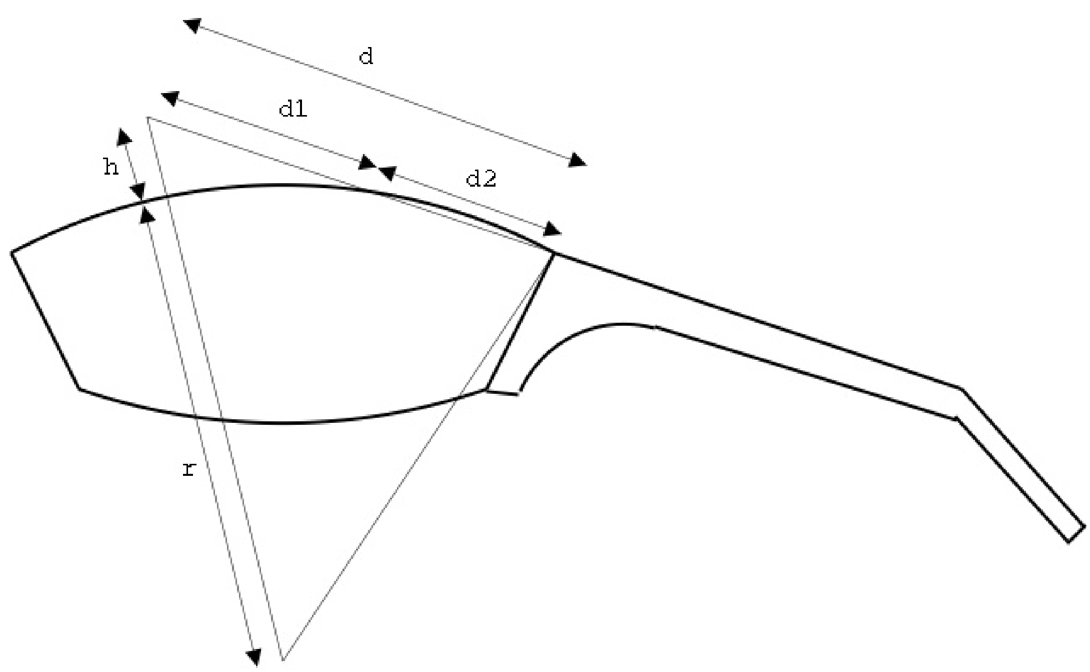

Figure 4

When a flat is planed on the top, the neck line becomes a secant to the top arc, rather than a tangent. Figure 4 shows that the height of the neck line above the top at the bridge position is less than in the simplified tangent case above. The height now depends not only on the distance from the neck-body junction to the bridge, but also on the amount that the neck line cuts through the arc, i.e., on the size of the flat planed for the fretboard. With this distance labeled as d_2 and the remaining distance labeled as d_1, the bridge height can now be derived as

$$h = \sqrt{r^2 + d_1^{\,2} + d_1 d_2} - r = r*\left[\sqrt{1 + \frac{d_1^{\,2}}{r^2} + \frac{d_1 d_2}{r^2}} - 1\right]$$

(The derivation is below.) If we write $d_1 = a*d$, i.e., we write d1 as some fraction of the total distance d, then d_2 becomes (1 - a)*d, and the formula simplifies to

$$h = r*\left[\sqrt{1 + a*\frac{d^2}{r^2}} - 1\right]$$

This looks just like the formula for the simplified case, except for the fractional multiplier a. In fact, the simplified tangent scenario is just the limiting case of the secant scenario when the size of the fret flat becomes 0, i.e., when d_1 = the entire distance d, which implies that the fractional multiplier a = 1. But the case of a = 1 gives us the previous formula. (And at the other extreme, if we were to plane the fret flat all the way to the bridge location, then we'd have the length of the fret flat being the entire distance d, i.e., $d_2 = d$, and $d_1 = 0$, or a = 0. But using a = 0 in the formula gives h = 0, which makes sense: the neck line passes right through the bridge position.)

As above, the preceding formula can be expressed in terms of the scale length L, using d = .44545 * L for a 14-fret neck or d = .5 * L for a 12-fret neck. For this formula, we also need to know the length of the fretboard flat planed onto the top, which is d_1 in the above formula. For my guitars, I use a 4" diameter soundhole whose center is 5 7/8" from the neck-body junction. I run the fretboard up to the edge of the soundhole, so the length of the fretboard flat is thus 3 7/8". The tables below thus tabulate the height of the neckline at the bridge location for various scale lengths and top arch radii using a fretboard flat of 3 7/8". As above, note that these heights don't include the fretboard thickness or fret heights - it's again just the additional height at the bridge that's the result of the top curving away from the strings.

Neck joined at 14th fret, fretboard flat 3.875": height at bridge in inches

	Top arch radius:				
Scale length:	**180" (15')**	**240" (20')**	**300" (25')**	**336" (28')**	**360" (30')**
24.5"	.21	.16	.13	.11	.11
24.75"	.22	.16	.13	.12	.11
25.0"	.22	.17	.13	.12	.11
25.25"	.23	.17	.14	.12	.12
25.5"	.24	.18	.14	.13	.12
25.75"	.24	.18	.15	.13	.12
26.0"	.25	.19	.15	.13	.12

Neck joined at 12th fret, fretboard flat 3.875": height at bridge in inches

	Top arch radius:				
Scale length:	**180" (15')**	**240" (20')**	**300" (25')**	**336" (28')**	**360" (30')**
24.5"	.28	.21	.17	.15	.14
24.75"	.29	.22	.18	.16	.15
25.0"	.30	.22	.18	.16	.15
25.25"	.31	.23	.18	.16	.15
25.5"	.31	.24	.19	.17	.16
25.75"	.32	.24	.19	.17	.16
26.0"	.33	.25	.20	.18	.16

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Derivation of the Secant Formula

Figure 5 illustrates the quantities needed for the derivation.

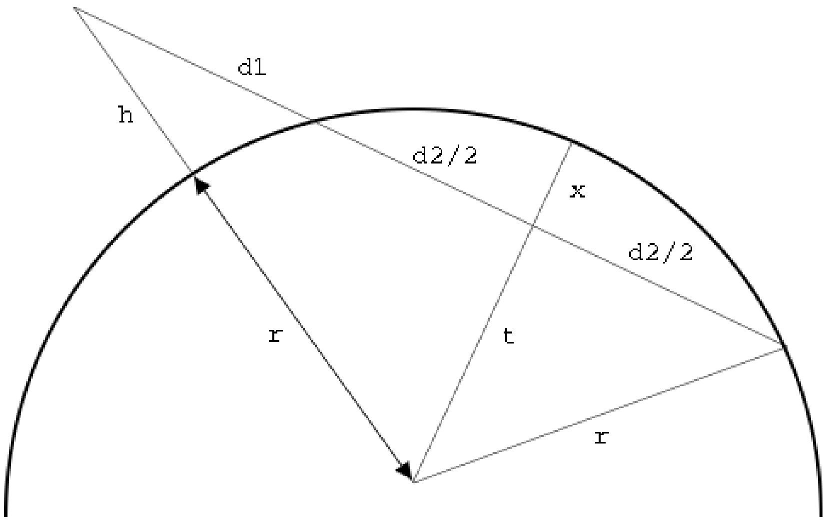

Figure 5

The line with segments t and x bisects the secant of length d_2, and thus is perpendicular to the secant. Thus both triangles in the figure are right triangles.

Applying the Pythagorean theorem to the triangle with sides of length t, $d_2/2$, and r gives

$$t^2 = r^2 - (d_2/2)^2$$

Applying Pythagoras to the larger triangle gives

$$t^2 = (r+h)^2 - (d_1+d_2/2)^2$$

Equating the t-squared in each equation gives

$$r^2 - (d_2/2)^2 = (r+h)^2 - (d_1+d_2/2)^2$$

Rearranging and expanding and simplifying gives

$$(r+h)^2 = r^2 - (d_2/2)^2 + (d_1+d_2/2)^2 = r^2 + d_1^2 + d_1 d_2$$

Taking the square root and solving for h gives the desired result:

$$h = \sqrt{r^2+d_1^2+d_1 d_2} - r$$